CKM matrix elements from Lattice QCD:
How well can we do?

C. Bernard

Washington University,

St. Louis, MO USA

Source

- Planning document for DoE: estimates of size of lattice errors attainable with given levels of computational resources.
- Meant to be an update to our original SciDAC proposal.
- Steve Sharpe and Bob Sugar took lead roles in preparing the document.
- Input from CB, Norman Christ, Aida El-Khadra, and Paul Mackenzie.

Outline

- Current lattice errors on CKM elements
- Lattice ensembles possible with given amounts of resources.
- Estimates of attainable errors
  - B_K
  - $f_{B_d}\sqrt{B_{B_d}}$ and ξ
  - $b \to u$ semileptonic form factors
  - $b \to c$ semileptonic form factors
- Summary table & plot
- Disclaimer

Current lattice errors

ϵ_K: CP violation in K_0–$\bar{K}_0$ mixing

$$|\epsilon_K| = C_\epsilon A^2 \lambda^6 \bar{\eta} \left[\eta_2 S(x_t) A^2 \lambda^4 (1-\bar{\rho}) + \mathrm{charm\!-\!contribs} \right] \hat{B}_K$$

- Use this to constrain $\bar{\rho}$ and $\bar{\eta}$
- Total non-lattice error is 9.5%, primarily from 2.2% error in $A\lambda^2 = |V_{cb}|$.
- Lattice $\hat{B}_K$ result used in recent fits is:

$$\hat{B}_K = 0.86 \pm 0.06 \pm 0.14 \qquad [\approx 18\% \text{ error}]$$

- $\therefore$ current lattice error is twice as large as error from other sources
- $\hat{B}_K$ milestones:
  - 10% error ($\approx$ that of other sources)
  - 5% error ($\approx$ half that of other sources)

Current lattice errors

ΔM_d: mass difference in B_d–$\bar{B}_d$ mixing

$$\Delta M_d = \frac{G_F^2 M_W^2 M_{B_d}}{6\pi^2} \eta_c S(x_t) A^2 \lambda^6 \left[(1-\bar{\rho})^2 + \bar{\eta}^2 \right] f_{B_d}^2 \hat{B}_{B_d}$$

- Total non-lattice error is 6% (from $A\lambda^2$, λ, η_c and ΔM_d in that order)
- Lattice result used in recent fits is:

$$f_{B_d}\sqrt{B_{B_d}} = 223 \pm 33 \pm 12 \text{ MeV} \qquad [\approx 15\% \text{ error}]$$

- Current lattice error is ≈ 5 times error from other sources ($f_{B_d}^2 B_{B_d}$ is relevant quantity)
- $f_{B_d}\sqrt{B_{B_d}}$ milestones:
  - $\sim 8\%$ error (reduction by factor of 2)
  - 3–4% error (comparable to that of other sources)

Current lattice errors

ΔM_s: mass difference in B_s–$\bar{B}_s$ mixing

$$\frac{\Delta M_d}{\Delta M_s} = \frac{M_{B_d}}{M_{B_s}} \lambda^2 \left[(1-\bar{\rho})^2 + \bar{\eta}^2 \right] \frac{1}{\xi^2}$$

$$\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$$

- ΔM_s not yet measured, but probably will be soon at Tevatron
- Present standard lattice result:

$$\xi = 1.24 \pm 0.04 \pm 0.06 \qquad [\approx 6\% \text{ error}]$$

- ξ milestones:
  - 3% error (reduction by factor of 2)
  - 1.5% error (reduction by factor of 4)

Current lattice errors

V_{ub}: from $b \longrightarrow u$ exclusive decays

- Current standard results don't use lattice:

$$|V_{ub}| = [33.0 \pm 2.4 \pm 4.6] \times 10^{-4} \qquad [\approx 16\% \text{ error}]$$

- Theory error ($4.6 \times 10^{-4} = 14\%$) from comparing models
- Current quenched lattice $B \to \pi\ell\nu$ form factors errors $\approx 15\%$
- $B \to \pi\ell\nu$ milestones:
  - 7% error ($\approx$ experimental error)
  - 3% error ($\approx$ half that of other sources)

Current lattice errors

V_{cb}

- Best current results use inclusive decays, not exclusive ones amenable to lattice treatment:

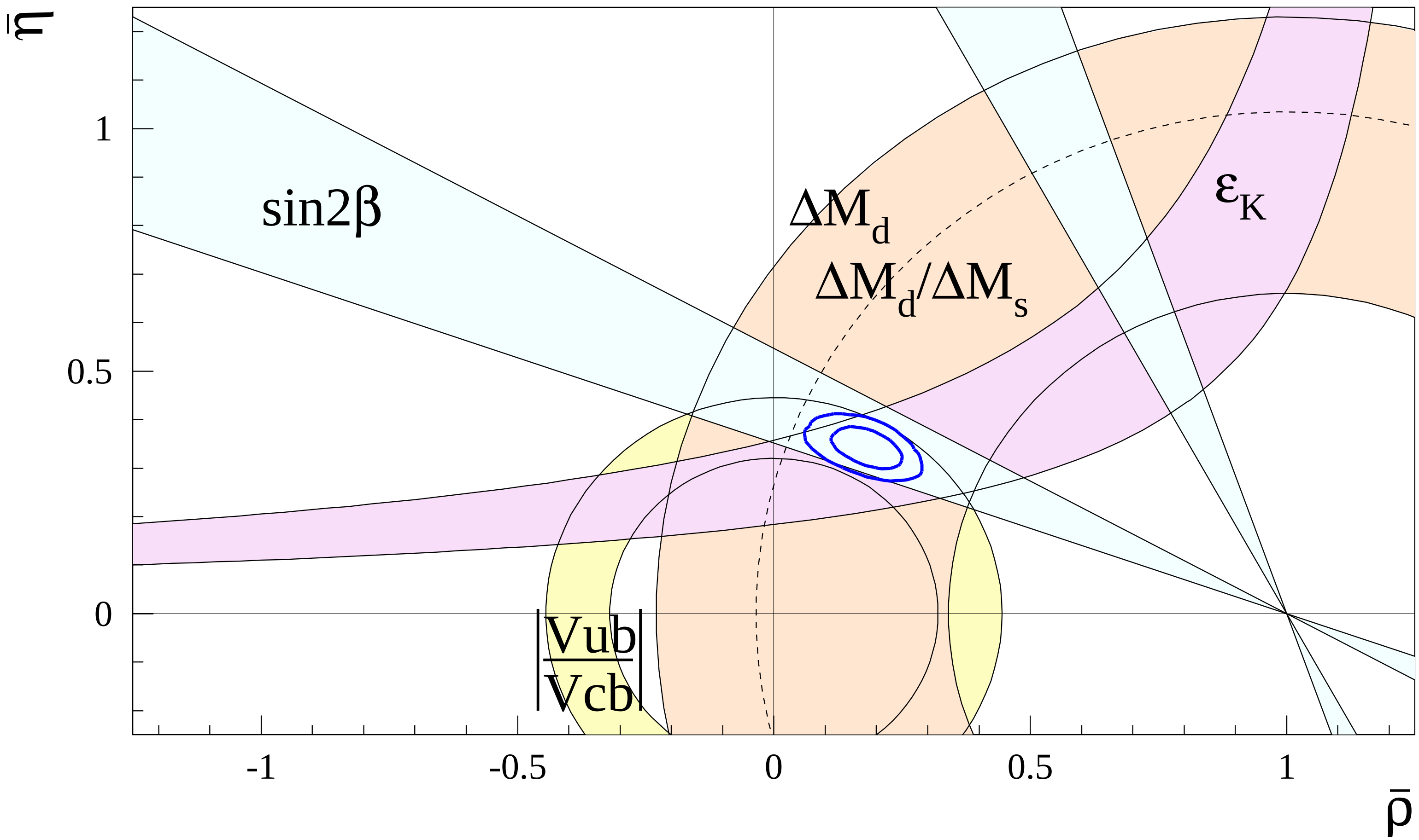
$$|V_{cb}| = [41.4 \pm 0.7 \pm 0.6] \times 10^{-3} \qquad [\approx 2.2\% \text{ error}]$$

- Theory error: $0.6 \times 10^{-3} = 1.4\%$
- Exclusive $B \to D^* \ell \nu$ result does use lattice:

$$|V_{cb}| = [42.1 \pm 1.1 \pm 1.9] \times 10^{-3} \qquad [\approx 5\% \text{ error}]$$

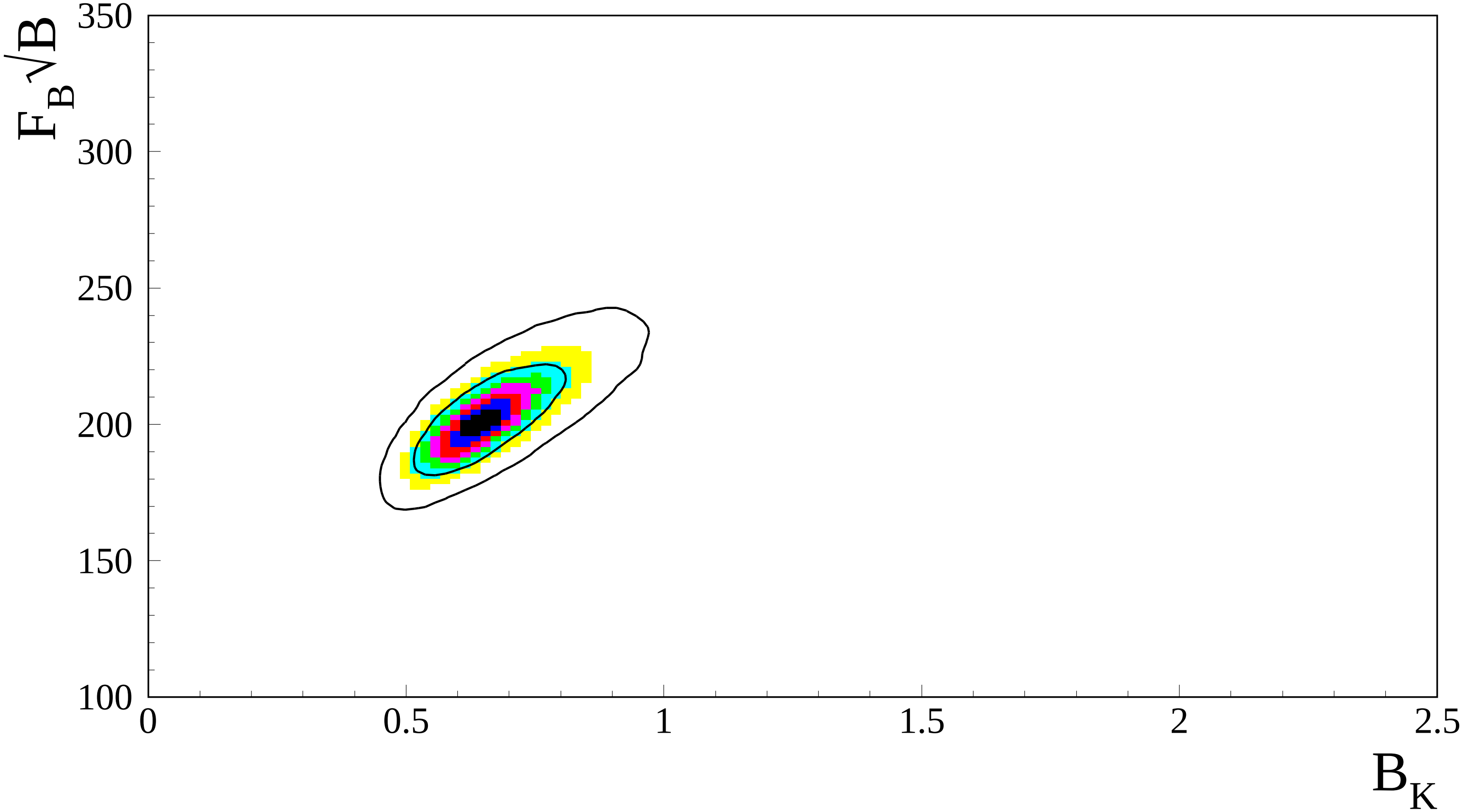
- Current Lattice error: $1.9 \times 10^{-3} = 4.5\%$
- $B \to D \ell \nu$, $B \to D^* \ell \nu$ milestones:
  - 2.5% error ($\approx$ exclusive experimental error)
  - 1.5% error ($\approx$ inclusive experiment or theory errors)

Current lattice errors: summary



Allowed regions for $\bar{\rho}$ and $\bar{\eta}$. 68% and 95% contours are shown. Full lines are 95% probability constraints from $|V_{ub}|\,/\,|V_{cb}|$, ϵ_K, ΔM_d and sin2β. Dotted curve bounds the 95% region from lower limit on ΔM_s.

Current lattice errors: summary



Allowed range of $\hat{B}_K$ and $f_{B_d}\sqrt{B_{B_d}}$, when taken as outputs of CKM fits, is comparable to lattice errors.

68% and 95% contours are shown. Fits use the $|V_{ub}|\,/\,|V_{cb}|$, Δm_s and sin2β constraints.

Possible Lattice Ensembles

Need to control continuum and chiral extrapolations

- Push to smaller lattice spacing a & smaller quark mass m_l
- Staggered quarks are fast — likely to dominate in near term
  - Current “fine” MILC ensemble: “MILC0” ($m_l/m_s = 0.2$, $a = 0.09$ fm)
  - Halve a^2 OR quark mass: “MILC1”
  - Halve a^2 AND quark mass: “MILC2”
- Staggered baggage:
  - Theoretical uncertainty introduced by $\sqrt[4]{\mathrm{Det}}$
  - Practical issue: taste violations mix continuum & chiral extraps; need staggered chiral perturbation theory (SχPT)

Possible Lattice Ensembles

- As resources increase, domain-wall (or overlap) dynamical fermions become more & more attractive
- Exact or near exact chiral symmetry $\Rightarrow$ continuum & chiral extrapolations decoupled.
- Consider ensemble "DWF1"
  - m_l and a^2 comparable to MILC1
  - Computer time comparable to MILC2
  - Expect errors comparable to MILC2 (because of separation of continuum & chiral limits)
  - Remove uncertainty from $\sqrt[4]{\mathrm{Det}}$
- Improved Wilson or twisted mass QCD are intermediate in resource requirements; may be especially useful in intermediate stages.

Timing Estimates

- Improved staggered
  - Time for MILC0 known
  - Time $\propto m_l^{-2.5}$ at fixed a and L: m_l^{-1} for D^{-1}; m_l^{-1} for step size; $m_l^{-0.5} \propto m_\pi$ for trajectory length
  - Time $\propto a^{-7}$ at fixed L and m_l: a^{-4} for lattice points, one a^{-1} each for D^{-1}, step size, and trajectory length
  - Time $\propto L^4$ at fixed a and m_l: lattice points
- Domain wall
  - current RBC runs give benchmark
  - Not clear what size of fifth dimension (N_s) or number of conjugate gradient iterations will be needed
  - Roughly, pay N_s to $2N_s$ factor over improved staggered
  - Perhaps 12–24$\times$ cost of comparable MILCx ensembles

Timing Estimates for Improved Staggered

m_l/m_s	a (fm)	Size	L (fm)	Tfl-yrs	m_π/m_ρ	Label
0.20	0.09	$28^3 \times 96$	2.5	0.09	0.39	MILC0
0.10	0.09	$40^3 \times 96$	3.6	1.5	0.30	MILC1
0.05	0.09	$56^3 \times 96$	5.0	23	0.22	
0.20	0.06	$42^3 \times 138$	2.5	1.5	0.39	MILC1
0.10	0.06	$60^3 \times 138$	3.6	25	0.30	MILC2
0.05	0.06	$84^3 \times 138$	5.0	390	0.22	
0.20	0.045	$56^3 \times 192$	2.5	12	0.39	
0.10	0.045	$80^3 \times 192$	3.6	190	0.30	
0.05	0.045	$112^3 \times 192$	5.0	2950	0.22	

Estimates of computer time needed to generate 120 independent lattices.

Final Timing Estimates

Time required to generate and analyze ensembles:

- MILC0: ≈ 0.6 Tflop-yrs. Configurations exist now, and most of the analysis is in progress (various groups).
- MILC1: ≈ 6 Tflop-yrs. Accessible to QCDOC & planned large clusters.
- MILC2: ≈ 50–60 Tflop-yrs. Requires next generation of machines.
- DWF1: ~ 100 Tflop-yrs. Requires next generation of machines.
- Note: below, "MILC2" is used to mean "MILC2 and/or DWF1."

Attainable errors: $\hat{B}_K$

- MILC0 + one-loop matching:
  - State of art is JLQCD quenched result: 18% error. (Unimproved staggered, 5 values of a, $a_{\rm min} = 0.05$ fm.)
  - Improved staggered at 0.09 fm should give comparable discretization errors to JLQCD minimum a.
  - Only two a values in MILC0 $\Rightarrow$ continuum extrapolation error of $\sim 5\%$ (compared to JLQCD 1%).
  - But previous 14% quenching error is removed.
  - Chiral extrapolation error subleading for $\hat{B}_K$ (can sit at m_K but $m_s - m_l$ too small) $\Rightarrow$ $\sim 5\%$ error.
  - One-loop matching not yet done but straightforward. JLQCD estimated 5% error by comparing operator discretizations. Expected to be larger in MILC0 because coupling is larger: $\alpha_s \approx 0.3 \Rightarrow \sim 9\%$ error.
- Total MILC0 $\hat{B}_K$ error $\sim 12\%$; close to first milestone (10%).

Attainable errors: $\hat{B}_K$

- MILC1
  - Can reduce chiral and continuum extrapolation errors.
  - Should do combined chiral and continuum extrapolation using SχPT. (SχPT calculation for B_K doesn't yet exist, but is being done by Sharpe & students.)
  - Estimate total chiral + continuum extrap error $\sim 2.5\%$ (like f_{Bd} MILC0 estimate below).
  - One-loop matching error still $\sim 9\% \Rightarrow \sim 10\%$ total error.
  - Doing better requires non-perturbative or (automated!) two-loop matching. Two-loop $\Rightarrow \sim 3\%$ error; non-perturbative matching would probably have comparable errors because of inherent statistics and systematics.
- Total MILC1 $\hat{B}_K$ error $\sim 5\%$ (second milestone) if two-loop or nonperturbative matching done.

Attainable errors: $\hat{B}_K$

- MILC2
  - Further reduction of chiral and continuum extrapolation errors.
  - Approach error limit of two-loop or non-perturbative matching.
- Total MILC2 $\hat{B}_K$ error $\sim 3\%$.
- IF we dream about 3-loop matching, then $\sim 1\%$ becomes accessible.

Attainable errors: f_{B_d}

- MILC0 + "standard" Fermilab ($\mathcal{O}(a)$, $\mathcal{O}(1/M)$) heavy quark + one-loop matching:
  - "Benchmarks" are existing f_{B_d} calculations with Wilson/Clover light quarks, and existing MILC0 f_π, f_K calculations.
  - MILC0 f_π has errors of: $\sim 2\%$ scale; $\sim 1.5\%$ chiral + continuum extrapolations (using SχPT); $\sim 0.8\%$ statistics.
  - For f_{B_d}, expect:
    - $\sim 2\%$ scale error
    - $\sim 2.5\%$ light quark errors. (Includes light quark chiral and discretization errors. Assumes SχPT, which is being worked out by Aubin & CB.) Larger than for f_π because $g_{BB^*\pi}$ (or $g_{DD^*\pi}$) not known accurately.

Attainable errors: f_{B_d}

- $\sim 3\%$ statistical errors. (Heavy-lights have larger fluctuations than light-lights. Old Wilson/Clover light quark results had $\sim 3\times$ bigger statistical errors for heavy-lights than light-lights.)
- $\sim 3\%$ heavy-quark discretization errors. (From old Fermilab results + comparison with more recent JLQCD NRQCD results, where truncation errors are expected to be somewhat larger.)
- ~ 5–10% one-loop perturbative matching error. (10% is α_s^2, same as $\hat{B}_K$ estimate. Fermilab trick of using nonperturbative $\sqrt{Z_{qq}Z_{QQ}}$, and just computing $Z_{qQ}/\sqrt{Z_{qq}Z_{QQ}}$ perturbatively, may reduce errors. But efficacy of this trick when heavy & light quarks have different actions is unknown. Also nonperturbative $\sqrt{Z_{qq}Z_{QQ}}$ will have unknown statistical errors.)

- Total MILC0 f_{B_d} error ~ 7–11%. (Current error is $\gtrsim 12\%$.)

Attainable errors: f_{B_d}

- MILC1 + “standard” Fermilab ($\mathcal{O}(a)$, $\mathcal{O}(1/M)$) heavy quark + two-loop matching:
- MILC1 is not much improvement unless two-loop matching is done, so we assume it. Not a big stretch to imagine two-loop calculations with automated perturbation theory coming on line in a year or two.
  - Perturbative errors: 5–$10\% \to 1$–3%.
  - Light quark errors: $2.5\% \to 1.5\%$. (Reduced taste violations at smaller a; closer to chiral limit at smaller m_l)
  - Scale error: $2\% \to 1.5\%$. (Better control over calculations that set scale.)
  - Heavy quark errors: $3\% \to 2\%$. (Closer to continuum.)
  - Statistical errors: $3\% \to 2\%$. (More independent sets to fit.)
- Total MILC1 f_{B_d} error ~ 3.5–4.5%.

Attainable errors: f_{B_d}

- MILC2 + "standard" Fermilab ($\mathcal{O}(a)$, $\mathcal{O}(1/M)$) heavy quark + two-loop matching.
  - Perturbative errors stay at 1–3%.
  - Light quark errors: $1.5\% \to 1\%$.
  - Scale error: $1.5\% \to 1\%$.
  - Heavy quark errors: $2\% \to 1.5\%$. ("Standard Fermilab" has $\mathcal{O}(\alpha_s a)$ or $\mathcal{O}(\alpha_s^2 a)$ corrections; improved more slowly than light quark as a is reduced. $\mathcal{O}(a^2)$ Fermilab version would help.)
  - Statistical errors: $2\% \to 1\%$.
- Total MILC2 f_{B_d} error ~ 2.5–4%.

Attainable errors: $f_{B_d}\sqrt{B_{B_d}}$

- It's really $f_{B_d}\sqrt{B_{B_d}}$ that's important.
- B_{B_d} errors compared to those of f_{B_d}:
  - Light quark error somewhat smaller because chiral extrapolation less steep. (*e.g.*, MILC0: $2.5\% \to 2\%$.)
  - Statistical error larger because 4-quark operators fluctuate more. (*e.g.*, MILC0: $3\% \to 5\%$.)
  - Heavy quark discretization error smaller because B_{B_d} is a ratio. (*e.g.*, MILC0: $3\% \to 2\%$.)
  - Scale error negligible because B_{B_d} is dimensionless.
  - Perturbative error estimate is the same.
- Estimated total $f_{B_d}\sqrt{B_{B_d}}$ errors:
  - MILC0: 8%–13% (current is $\sim 15\%$)
  - MILC1: 4%–5% ($\sim$first milestone; two-loop matching is key)
  - MILC2: 3%–4% ($\sim$second milestone)

Attainable errors: $\xi \equiv f_{B_s}\sqrt{B_{B_s}}/(f_{B_d}\sqrt{B_{B_d}})$

- ξ errors compared to those of $f_{B_d}\sqrt{B_{B_d}}$:
  - Light quark error similar because chiral extrapolation not relevant to $f_{B_s}\sqrt{B_{B_s}}$.
  - Statistical error smaller because ξ is a ratio. (*e.g.*, MILC0: $4\% \to 2\%$.)
  - Heavy quark discretization error smaller because ξ is a ratio. (*e.g.*, MILC0: $3\% \to 1\%$.)
  - Scale errors small because ξ dimensionless. (But scale does enter through m_s.)
  - Perturbative errors in ξ almost completely cancel.
- Estimated total ξ errors:
  - MILC0: 4% (current is $\sim 6\%$)
  - MILC1: 3% (first milestone)
  - MILC2: 1.5%–2% ($\sim$ second milestone)

Attainable errors: $B \to \pi\ell\nu$ form factors

- $B \to \pi\ell\nu$ errors compared to those of f_{B_d}:
  - Light quark errors larger, based on Fermilab quenched calculations. Due to finite π momentum? (*e.g.*, MILC0: $2.5\% \to 5\%$.) (Assumes SχPT; in progress by Aubin & CB.)
  - Statistical error larger because of finite π momentum. (*e.g.*, MILC0: $3\% \to 4.5\%$.)
  - Heavy quark discretization error comparable.
  - Scale error small because form factors are dimensionless. (But scale still enters, *e.g.*, through normalization of momenta.)
  - Perturbative error estimate is the same.
- Estimated total $B \to \pi\ell\nu$ errors:
  - MILC0: 10%–13% (current is $\sim 15\%$)
  - MILC1: 5.5%–6.5% ($\lesssim$ first milestone; assume 2-loop)
  - MILC2: 4%–5% (close to second milestone)

Attainable errors: $B \to D^{(*)}\ell\nu$ form factors

- $B \to D^{(*)}\ell\nu$ errors compared to those of f_{B_d}:
  - Using Fermilab ratio method, errors scale with $\mathcal{F} - 1$, where $\mathcal{F}$ is endpoint form factor.
  - Expect statistical, perturbative, heavy quark errors to be about $1/3$ as large as for f_{B_d}.
  - Light quark errors are similarly suppressed, but it was not assumed that SχPT would exist for this case. So assumed light quark errors of $2/3$ as large as for f_{B_d}.
  - Note: now seems like SχPT will not be a problem, but nobody yet doing this calculation, as far as I know.
  - Estimated total $B \to D^{(*)}\ell\nu$ errors:
    - MILC0: 3%–4% (current is$\sim 4.5\%$)
    - MILC1: 1.8%–2% ($\sim$first milestone; assume 2-loop)
    - MILC2: 1%–1.4% ($\lesssim$second milestone)

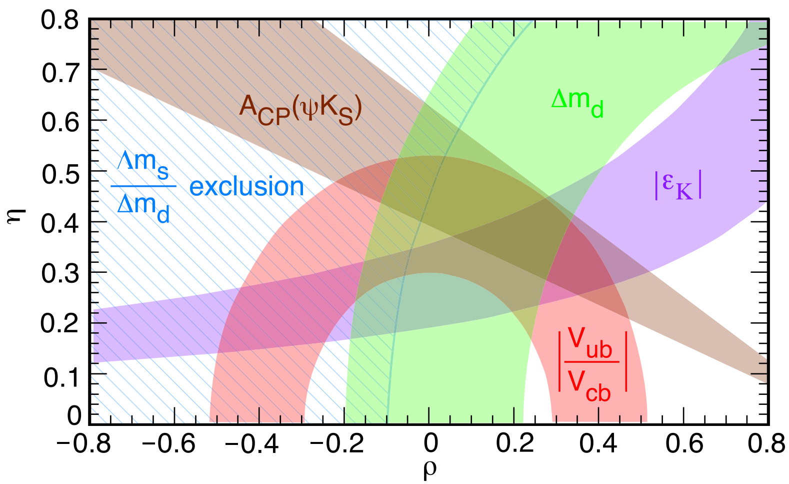
Summary Table

Measurement	CKM Matrix Element	Hadronic Matrix Element	Non-Lattice Errors	Current Lattice Errors	Lattice Errors 0.6 TF-Yr MILC0	Lattice Errors 6.0 TF-Yr MILC1	Lattice Errors 60. TF-Yr MILC2/ DWF1
ε_K ($\bar{K}K$ mixing)	$\mathrm{Im}\, V_{td}^2$	$\hat{B}_K$	10%	20%	12%	5%	3%
ΔM_d ($\bar{B}B$ mixing)	$\lvert V_{td}\rvert^2$	$f_{B_d}^2 B_{B_d}$	6%	30%	16%–26%	8%–10%	6%–8%
$\Delta M_d / \Delta M_s$	$\lvert V_{td}/V_{ts}\rvert^2$	ξ^2	—	12%	8%	6%	3%–4%
$B \to \pi \ell \nu$	$\lvert V_{ub}\rvert^2$	$\langle \pi \rvert (V-A)_\mu \lvert B \rangle$	7%	15%	10%–13%	5.5%–6.5%	4%–5%
$B \to \binom{D^*}{D} \ell \nu$	$\lvert V_{cb}\rvert^2$	$\lvert \mathcal{F}_{B\to\binom{D^*}{D}\ell\nu}\rvert^2$	2%	4.4%	3%–4%	1.8%–2%	1%–1.4%

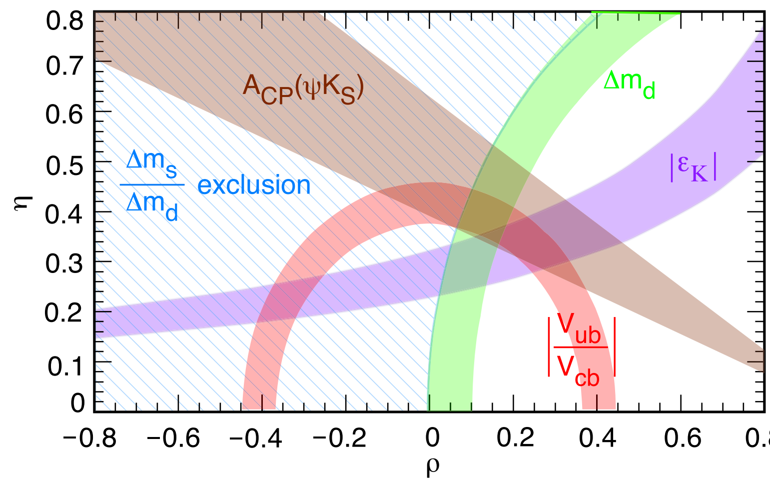
(Up to minor modifications, this table is same as that presented to HEPAP by Bob Sugar.)

Impact of Reduced Lattice Errors

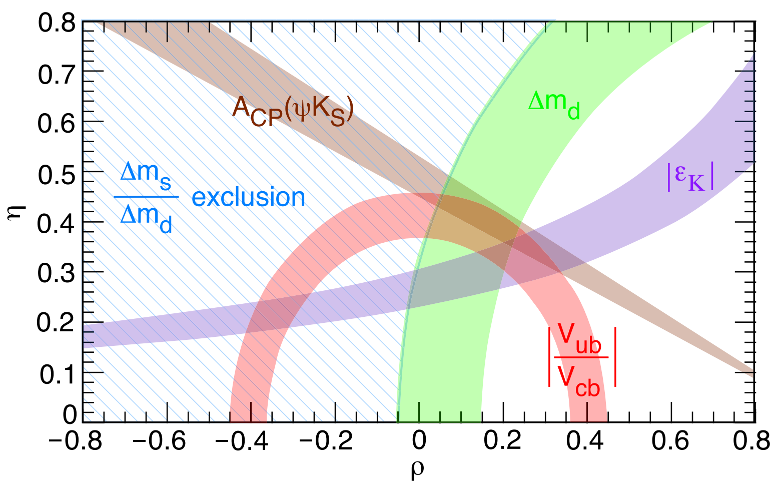
CKM today . . .

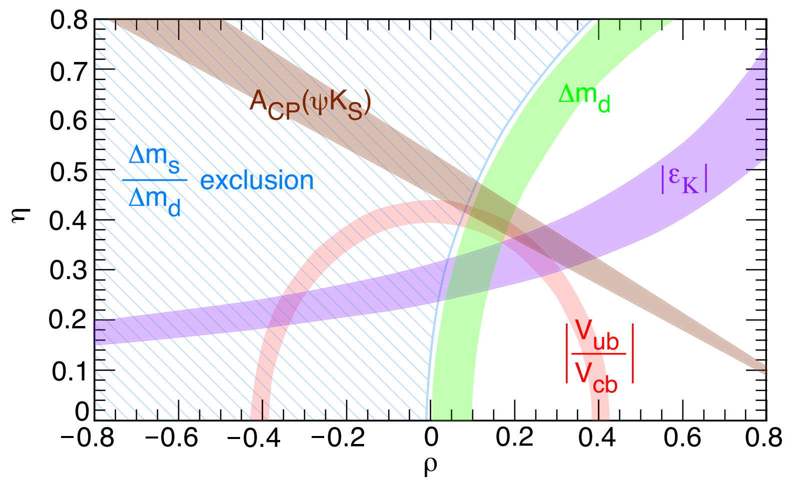


. . . and with 2–3% theory errors.



And with B Factories . . .





The impact of the B factories and improvements in lattice calculations on parameters of the CKM matrix. CLEO-c Collaboration (2001).

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- Steve Sharpe called it a “Fool’s Errand.”
- He was not available to give this talk; suggested me instead.